5 Module E: The Fourier Transform

We have already encountered various forms of the relationship between the and frequency domains. In this module, we explore the Fourier Transform and its inverse, which transform a signal between its time-domain representation and its frequency-domain representation. There are four versions, depending on whether the time-domain signal is continuous or discrete, and on whether it is periodic or aperiodic. We will be focusing on the discrete-time versions.

Note: this module does not cover the whole of Chapter 10 of Lee and Varaiya.

5.1 Readings and other resources

The following readings are taken from the textbook by Lee and Varaiya. You can use the questions in the following section to help you find your way through these readings.

Readings

- Lee and Varaiya, Chapter 10 intro
- Lee and Varaiya, Section 10.1, *Notation*
- Lee and Varaiya, Section 10.2, *The Fourier series, up to p 372*
- Lee and Varaiya, Section 10.3, *The discrete Fourier transform*
- Lee and Varaiya, Section 10.4, *The discrete-time Fourier transform*
- Lee and Varaiya, Section 10.6, *Fourier transforms versus Fourier series*

On-line resources

- Generate some interesting sounds to analyze (Java applet)
  http://www.cs.ubc.ca/spider/kvdoel/sound_demo6.html

5.2 Text-reading questions

The "answers" to these questions are obtained readily from the required readings listed above, in the section indicated in parentheses. The purpose of working through these question is to ensure that you have read the important parts of the text, and also to serve as a useful reference for you for later study.

1. What is the relationship between the Fourier series (first introduced in Module C) and the Fourier transform? (10.intro)

2. For the purposes of this module, what is domain and range are used for continuous and discrete signals? (10.1)

3. Explain the difference between a periodic signal and an aperiodic signal. (10.1, see also section 7.5.2)

4. Write down, and then explain in words, the mathematical equation for the Fourier series coefficients X_m. (10.2)

5. What is the domain and range of the Fourier Series version of the Fourier transform? Explain, with the aid of a diagram, the nature of the input and output signals. (10.2)

6. What is an *inverse* Fourier transform? Give examples. (10.2)

7. Write the equation for the discrete Fourier transform (DFT), and explain it in words. (10.3)

8. Write the equation for the *inverse* discrete Fourier transform (IDFT), and explain it in words. (10.3)

9. Derive the equation for the frequency domain coefficients of a discrete-time square wave with period 6. (10.3)

10. Write the equation for the discrete-time Fourier transform (DTFT), and explain it in words. (10.4)

11. Explain the difference between the DFT and the IDFT. (10.4)

12. Write the equation for the *inverse* discrete-time Fourier transform (IDTFT), and explain it in words. (10.4)

13. Explain the difference between the IDFT and the IDTFT. (10.4)

14. Derive the equation for the frequency domain coefficients of a discrete-time pulse of length 3. (10.4)

5.3 Laboratory exercises

5.3.1 Lab E1

In this lab, you will be exercising some basic Fourier transform theory.

1. Write a Matlab function, *mydft*, that computes the DFT of its argument, x. To do this, you simply need to implement the equation

$$\forall k \in \text{Integers}, X_k = \sum_{m=0}^{p-1} x(m) e^{-jmk\omega_0}$$

2. Test *mydft* by using it to calculate the Fourier transform of the following discrete periodic waveforms, using a period of 32 samples. Plot the amplitude and phase components of each result, and verify that you get the same results as using Matlab's inbuilt *fft* function. Are the results what you would have expected from theory?

   (a) A cosine wave at the fundamental frequency

   (b) A cosine wave at twice the fundamental frequency

   (c) A sine wave at the fundamental frequency

   (d) A square wave

   (e) An impulse train, with 4 samples between impulses

   (f) The same impulse train, delayed by one sample

5.3.2 Lab E2

—To be provided during semester—

5.4 Self-study questions

1. Use Matlab to plot the Fourier transform of the following *discrete periodic* waveforms. (We suggest you try using a period of 32 samples.) Plot both the amplitude and phase components.

   (a) A square wave
   (b) A cosine wave
   (c) A sine wave
   (d) An impulse train
   (e) An impulse train delayed by one sample

2. Use Matlab to plot the Fourier transform of the following *discrete aperiodic* signals. Plot both the amplitude and phase components. Note: explain how you are approximating the result.

   (a) An impulse
   (b) An impulse delayed by N samples (try value of N from 1 to 4)
   (c) A rectangle pulse
   (d) A single cycle of a sinusoid

3. Use Matlab to plot the Fourier transform of the following *continuous periodic* waveforms. Plot both the amplitude and phase components. Note: explain how you are approximating the continuous-time waveform.

   (a) A 400 Hz sinusoid
   (b) A 440 Hz sinusoid added to a 400 Hz sinusoid
   (c) A 100 Hz square wave

4. Use Matlab to plot the Fourier transform of the following *continuous aperiodic* waveforms. Plot both the amplitude and phase components. Note: explain how you are approximating the continuous-time waveform and frequency response.

   (a) An impulse
   (b) An impulse delayed by τ seconds (try a few different values of τ)
   (c) A rectangle pulse
   (d) The function $sin(\pi x)/(\pi x)$

5. For the signals in the previous four questions, compare your result obtained using Matlab to the result that would be expected using analytical methods.

6. Lee and Varaiya, Chapter 10, question 5.

7. Lee and Varaiya, Chapter 10, question 6.

8. Lee and Varaiya, Chapter 10, question 7.

9. Lee and Varaiya, Chapter 10, question 8.

10. Lee and Varaiya, Chapter 10, question 18.