4 Module D: Filtering

Recall that the output signal of a linear system contains only the frequencies present in the input signal. Linear systems are thus often referred to as *filters*, since they alter the amplitude (and phase) of the frequency components of the input signal, but without adding any additional components. Filters are a basic tool in almost all signal-processing systems, and there is a huge literature in ways to design filters to suit a particular specialized application. In this module, we will explore the basics of filters and their design.

4.1 Readings and other resources

The following readings are taken from the textbook by Lee and Varaiya. You can use the questions in the following section to help you find your way through these readings.

Readings

- Lee and Varaiya, Chapter 9 intro
- Lee and Varaiya, Section 9.1, *Convolution*
- Lee and Varaiya, Section 9.2, *Frequency response and impulse response*
- Lee and Varaiya, Section 9.3, *Causality*
- Lee and Varaiya, Section 9.4, *Finite impulse response filters*
- Lee and Varaiya, Section 9.5, *Infinite impulse response filters*

On-line resources

- Filtering sounds (Java applet)
  http://ptolemy.eecs.berkeley.edu/eecs20/week10/sounds.html
- Compares the effect of different filters (Java applet)
  http://www.falstad.com/dfilter/
- The Joy of Convolution (Java applet)
  http://www.jhu.edu/ signals/discreteconv2/index.html
- Convolution revisited (Java applet)
  http://ptolemy.eecs.berkeley.edu/eecs20/week11/convolutionRA.html

4.2 Text-reading questions

The "answers" to these questions are obtained readily from the required readings listed above, in the section indicated in parentheses. The purpose of working through these question is to ensure that you have read the important parts of the text, and also to serve as a useful reference for you for later study.

1. What is a filter (and why)? (9.intro)

2. What is an equalizer and why is it used? (9.intro)

3. Write the equation for the (discrete-time) convolution sum. (9.1)

4. If x is an impulse, and y is the impulse response of a four-point moving average filter, draw a graphical representation of the convolution sum for $n = 2$. (9.1)

5. What are the key properties of convolution? (9.1)

6. Write the equation for the (continuous-time) convolution integral. (9.1)

7. Define, compare, and contrast the discrete-time and continuous-time impulse functions (9.1)

8. Explain, both mathematically, and graphically, how a discrete-time signal can be composed of a sum of impulses. (9.1)

9. Explain mathematically why the output of a system is equal to the convolution of the input signal with the system's impulse response. (9.1)

10. What is the impulse response of two systems connected in series (cascade)? Verify this diagrammatically by cascading a 4-point "echo" with a two-point moving average filter. (9.1)

11. Derive the equation relating the impulse response of a system to its frequency response. (9.2)

12. What is a causal system? Why is causality a significant property of a system? (9.3)

13. Rewrite the equation for the convolution sum if the impulse response is finite. (9.4)

14. What is a passband? What is a stopband? A transition band? (9.4)

15. A system that simply multiplies its input by a factor of 10 has what gain (in dB)? What about a system that multiplies its input by 0.01? (9.4)

16. What is the multiplying factor for the following gains (in dB): 100 dB; 14 dB; 6 dB; 3 dB; -26 dB. (9.4)

17. Give the general difference equation for an LTI system with an infinite impulse response. Write the value of the a and b coefficients for: a) the four-point moving-average filter; b) a unit delay; c) the simple recursive filter from Module B; and d) an "echo" with a delay of 4. (9.5)

18. What is a unit step? (9.5)

19. Name several key types of IIR filter. In what ways do these filters differ? (Draw graphs to illustrate.) (9.5)

20. What is passband ripple? What is stopband ripple? (9.5)

4.3 Laboratory exercises

4.3.1 Lab D1

This lab aims to cement your understanding of the relationship between:

- A system's impulse response and frequency response
- The declarative and imperative descriptions of a system

1. Consider the system S with the following declarative description:

$$\forall n \in \text{Integer}_{+}, y(n) = 0.1x(n) + x(n-1) + x(n-2) + 0.1x(n-3)$$

   (a) Calculate the impulse response of S. (By now, you should be able to do this one by inspection.)

   (b) Plot the impulse response of S, with properly labeled axes.

2. Use the Fourier Transform to generate the frequency-domain representation of the impulse response. This is the frequency response of S. Use Matlab to plot the amplitude and phase response of S.

3. Generate a *sinusoidal* waveform at three different frequencies ω_1, ω_2, and ω_3. For each:
   (a) Use the Matlab function *conv* to convolve the sinusoidal waveform with the impulse response of S.
   (b) Examine the output signal to determine its amplitude and phase for each of ω_1, ω_2, and ω_3.

   Compare to your results from Question 2.

4. Use a state-space model to implement S.
   (a) Generate a state-space model of S (the $[A, B, C, D]$ matrices).

   (b) Use your state-space calculation function from earlier labs to verify that the impulse response of your state-space model is correct.

   (c) Use your state-space calculation function from earlier labs to compute the amplitude and phase response at each of the three frequencies ω_1, ω_2, and ω_3. Compare to your results from Question 3.

4.3.2 Lab D2

—To be provided during semester—

4.4 Self-study questions

1. (a) Explain in plain English -
    - what is an impulse response
    - how you can determine the impulse response of a system
    - what is a frequency response
    - how you can determine the frequency response of a system
    - the relationship between the impulse response and the frequency response

   (b) Using an example of a specific linear system, illustrate your answers to a) both mathematically and graphically.

2. Lee and Varaiya, Chapter 9, question 1
3. Lee and Varaiya, Chapter 9, question 4
4. Lee and Varaiya, Chapter 9, question 6
5. Lee and Varaiya, Chapter 9, question 7
6. Lee and Varaiya, Chapter 9, question 10
7. Lee and Varaiya, Chapter 9, question 13
8. Explain in plain English:

   (a) what information is contained in the frequency response of systems

   (b) why this information can be important in telecommunications

   (c) what is a filter, and how are they used